temperature measurements. As required for a weak interaction regime an inviscid region separates the shock wave from a constant pressure viscous layer where the pressure is equal to the surface pressure. This viscous layer pressure is also shown in Fig. 2 and is in very good agreement with the corrected surface pressure when an estimate is made for the effect of slip. In addition, the static pressure and temperature that rise across the shock wave agree with the Rankine-Hugoniot values to within 4%. Also, the freestream rotational temperatures agreed with the calculated static temperature to within $^{+2}_{-0}\%$. Although there is still some doubt about the secondary excitation process involved, these measurements demonstrate that Eq. (1), based on the experiments of Ashkenas, does not result in serious systematic errors over the temperature range 70°K to room temperature, at least for the density levels encountered in this experiment.

The profiles for the merged regime show that the static pressure increases across the viscous layer, the gradients becoming larger as the leading edge is approached. In this regime the shock wave is not of primary importance in determining the surface pressure, and the static pressure behind the shock will only equal the surface pressure at the point where merging occurs. Because of the increasing uncertainty in T_R as the surface is approached, it is dangerous to extrapolate these profiles to the surface, although the pressure profile 1.9 cm from the leading edge shows reasonable agreement with the curve of Bartz and Vidal + slip.

There are no known theoretical results with which these measurements can be compared directly. Oguchi[11] estimated that for a highly cooled wall the normal pressure gradient was small and this has been used in many theoretical models. Huang and Hwang[12] have computed profiles for $M = 5$ and $T_w/T_0 = 0.15$ which show considerable variations in static pressure across the viscous layer. The present results show that a constant pressure viscous layer model cannot be used for higher wall temperatures, and there is a need for further studies of this type to check Oguchi's estimate of a small normal pressure gradient for a highly cooled wall.

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Application of Whitham's Theory to Sonic Boom in the Mid- or Near-Field

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Introduction

In experimental investigations of sonic boom problems in wind tunnels, it is usually necessary to use very small models in order to get direct results for the far-field. With these methods, inaccuracies due to the model contours, boundary-layer development, etc., usually arise. A new method is based on large models in wind tunnels where only the near- or mid-field is simulated. The pressure signatures of the models in the far-field are calculated from the pressure distributions measured in the vicinity of the wind tunnel wall. The present investigation is directly related to this new method. That is, to extrapolate a known pressure signature in the near- or mid-field to the far-field.

Calculations of sonic boom pressure signatures of an aircraft have been based mainly on Whitham's theory,[1] which describes the asymptotic behavior of the flowfield far from an equivalent body of revolution of the aircraft. Investigators on sonic boom, however, have used this theory to predict nonasymptotic pressure signatures in a nearer field.[2,3] In the mid- or near-field, Whitham,[4] Lighthill,[5] and more recently, Moore and Henderson[2] have considered asymptotic expansions of Whitham's theory in order to obtain certain modifications. However, to the author's knowledge, no general quantitative modifications of Whitham's theory in the mid- or near-field have been obtained.

In this Note, we shall be concerned with the validity of Whitham's theory in the mid- or near-field of a slender body. We shall be particularly interested in the quantitative and qualitative modification of Whitham's theory and in the extrapolation of a known disturbance signature to the far field.

Asymptotic Relations of Whitham

For simplicity, we consider a steady, homogeneous, inviscid, supersonic flow over a slender body of revolution. It is clear that the disturbance of the flowfield due to the presence of a slender body is equivalent to that caused by a streamtube (i.e., a quasi-cylinder) enclosing this body in a nearer field. Therefore, instead of considering the flowfield of the body, we shall consider the flowfield over a streamtube enclosing this body.

Let us choose the axis of the body to be the x axis coinciding with the freestream direction (Fig. 1). Enclosing this body, we choose a coaxial circular streamtube with a radius R. On the streamtube surface, the flow starts to be disturbed at $x = 0$, where the origin of the x axis is defined. The r axis perpendicular to the x axis is the radial coordinate.

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Based on the linearized supersonic theory, the flow disturbances downstream of the streamtube can be expressed as integrals of the source distribution $f(x)$ of this streamtube. This source distribution $f(x)$ may be determined from the linearized boundary conditions on the streamtube surface. After knowing $f(x)$, we may define Whitham's F-function for the streamtube;

$$F(y) = \int_0^y \frac{f'(t - \alpha R)}{(y - t)^{1/2}} dt \qquad (1)$$

By following Whitham's hypothesis on improvement of characteristics and by applying his far-field approximation $y/\alpha r \ll 1$, we may obtain Whitham's asymptotic relations,

$$(1/\gamma M_\infty{}^2)\Delta p/p_\infty = -u = v/\alpha = F(y)/(2\alpha r)^{1/2} \qquad (2)$$

with $\alpha = (M_\infty{}^2 - 1)^{1/2}$. Here y is determined from the condition that $y(x,r;\ R) =$ constant is a characteristic curve,

$$x = \alpha(r - R) + y - kF(y)[r^{1/2} - R^{1/2}] \qquad (3)$$

with $k = 2^{-1/2}(\gamma + 1)M_\infty{}^4\alpha^{-3/2}$. The relations (2) and (3) are valid asymptotically at a distance sufficiently far from the streamtube. For a slender body, $R \rightarrow 0$, Eqs. (1–3) reduce exactly to those of Whitham.[1]

Modification in the Near- or Mid-Field

In this section, we shall examine the flowfield at the neighborhood of the streamtube. From the linearized boundary conditions, we shall find the source distribution $f(x)$ in terms of the flow disturbances on the streamtube surface. Then we shall derive the relationships between the flow disturbances and the F function of this streamtube.

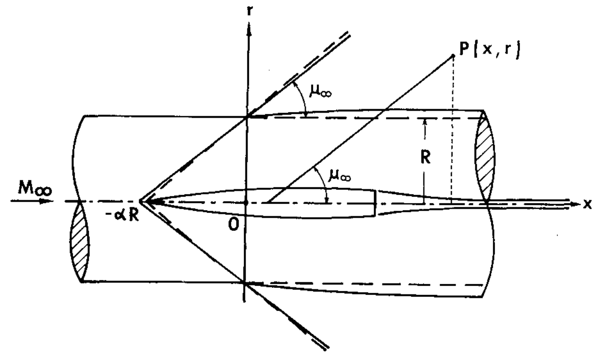

Fig. 1 Streamtube and coordinate system.

Let the flow disturbances be known on the streamtube surface; within the linearized supersonic theory,

$$\frac{1}{\gamma M_\infty{}^2}\frac{\Delta p}{p_\infty} = -u = P(x) = \int_{-\alpha R}^{x-\alpha R} \frac{f'(t)dt}{[(x - t)^2 - \alpha^2R^2]^{1/2}} \qquad (4)$$

and

$$v = R'(x) = \frac{1}{R}\int_{-\alpha R}^{x-\alpha R} \frac{(x - t)f'(t)dt}{[(x - t)^2 - \alpha^2R^2]^{1/2}} \qquad (5)$$

These are integral equations for $f'(t)$. Equation (5) was solved by Lighthill[6]; Eq. (4) may be solved similarly.

By changing the integrating variable, Eq. (4) becomes

$$P(x) = \int_0^x \frac{f'(t - \alpha R)dt}{[(x - t)(x - t + 2\alpha R)]^{1/2}} \qquad (6)$$

Let us nondimensionalize coordinate variables by αR (i.e., $\bar{x} = x/\alpha R$ and $\bar{t} = t/\alpha R$), and set $f'(t - \alpha R) = h(\bar{t})$ and $P(x) = g(\bar{x})$. Then, Eq. (6) reduces to

$$g(\bar{x}) = \int_0^{\bar{x}} \frac{h(\bar{t})d\bar{t}}{[(\bar{x} - \bar{t})(\bar{x} - \bar{t} + 2)]^{1/2}} \qquad (7)$$

This is an integral equation of the first kind of Volterra,[7] and it may be solved by Lighthill's procedure.[6] Define

$$b(\bar{s}) = \int_0^{\bar{s}} \frac{(\bar{x})d\bar{x}}{(\bar{s} - \bar{x})^{1/2}} \qquad (8)$$

then

$$b(\bar{s}) = \frac{\pi}{2^{1/2}}\int_0^{\bar{s}} K(\bar{s} - \bar{t})h(\bar{t})d\bar{t} \qquad (9)$$

with

$$K(z) = \frac{2}{\pi}\left(\frac{2}{2 + z}\right)^{1/2} F_1\left[\left(\frac{z}{2 + z}\right)^{1/2}\right] \qquad (10)$$

where F_1 is the complete elliptic integral and $K(0) = 1$. Hence, we can reduce Eq. (9) to an integral equation of the second kind by differentiation,

$$b'(\bar{s}) = \frac{\pi}{2^{1/2}}\left[h(\bar{s}) + \int_0^{\bar{s}} K'(\bar{s} - \bar{t})h(\bar{t})d\bar{t}\right] \qquad (11)$$

Let $(2^{1/2}/\pi)b'(\bar{s}) = w(\bar{s})$; the solution of Eq. (11) is

$$h(\bar{t}) = w(\bar{t}) - \int_0^{\bar{t}} S(\bar{t} - \bar{s})w(\bar{s})d\bar{s} \qquad (12)$$

where S is the resolvent of K'.[7] The functions $K'(z)$ and $S(z)$ are tabulated in Table 1 for $z =$ 0–10.

Table 1 Functions of $K'(z)$, $S(z)$, $K_1'(z)$ and $S_1(z)$

z	$K'(z)$	$S(z)$	$K_1'(z)$	$S_1(z)$
0.0	−0.12500	−0.12500	0.37500	0.37500
0.1	−0.11832	−0.11980	0.36377	0.35038
0.2	−0.11227	−0.11510	0.35344	0.32793
0.3	−0.10676	−0.11083	0.34389	0.30738
0.4	−0.10174	−0.10692	0.33504	0.28854
0.5	−0.09714	−0.10334	0.32680	0.27123
0.6	−0.09290	−0.10004	0.31911	0.25529
0.7	−0.08899	−0.09699	0.31192	0.24059
0.8	−0.08538	−0.09416	0.30157	0.22702
0.9	−0.08203	−0.09153	0.29882	0.21443
1.0	−0.07891	−0.08907	0.29283	0.20278
1.2	−0.07328	−0.08462	0.28182	0.18190
1.4	−0.06835	−0.08069	0.27193	0.16381
1.6	−0.06399	−0.07718	0.26297	0.14808
1.8	=0.06012	−0.07404	0.25481	0.13430
2.0	−0.05666	−0.07120	0.24735	0.12221
2.2	−0.05354	−0.06861	0.24048	0.11156
2.4	−0.05072	−0.06625	0.23414	0.10212
2.6	−0.04816	−0.06408	0.22827	0.09375
2.8	−0.04583	−0.06208	0.22280	0.08630
3.0	−0.04370	−0.06023	0.21770	0.07963
3.2	−0.04173	−0.05850	0.21293	0.07367
3.4	−0.03993	−0.05690	0.20845	0.06829
3.6	−0.03826	−0.05539	0.20423	0.06346
3.8	−0.03671	−0.05398	0.20026	0.05910
4.0	−0.03527	−0.05265	0.19650	0.05516
4.4	−0.03269	−0.05021	0.18957	0.04832
4.8	−0.03042	−0.04803	0.18332	0.04263
5.2	−0.02843	−0.04605	0.17763	0.03786
5.6	−0.02666	−0.04426	0.17244	0.03384
6.0	−0.02508	−0.04262	0.16767	0.03042
6.4	−0.02366	−0.04111	0.16327	0.02749
6.8	−0.02238	−0.03971	0.15919	0.02498
7.2	−0.02122	−0.03842	0.15540	0.02280
7.6	−0.02016	−0.03721	0.15187	0.02089
8.0	−0.01917	−0.03609	0.14857	0.01921
8.4	−0.01831	−0.03504	0.14547	0.01776
8.8	−0.01749	−0.03405	0.14255	0.01645
9.2	−0.01674	−0.03312	0.13980	0.01530
9.6	−0.01605	−0.03224	0.13720	0.01426
10.0	−0.01540	−0.03141	0.13475	0.01333

The integral Eq. (5) was solved by Lighthill,[6] and its solution is

$$h(\bar{t}) = w_1(\bar{t}) - \int_0^{\bar{t}} S_1(\bar{t} - \bar{s})w_1(\bar{s})d\bar{s} \tag{13}$$

Here,

$$w_1(\bar{t}) = \frac{2^{1/2}}{\pi}\left[\frac{g_1(0)}{\bar{t}^{1/2}} + \int_0^{\bar{t}} \frac{g_1'(\bar{x})d\bar{x}}{(\bar{t} - \bar{x})^{1/2}}\right] \tag{14}$$

with $g_1(\bar{x}) = R'(x)/\alpha$, and S_1 the resolvent for $K_1'(z)$. $K_1'(z)$ is the differentiation of $K_1(z)$, and

$$K_1(z) = \frac{2^{3/2}}{\pi}\left\{(2 + z)^{1/2}E_1\left[\left(\frac{z}{2 + z}\right)^{1/2}\right] - \frac{1}{(2 + z)^{1/2}}F_1\left[\left(\frac{z}{2 + z}\right)^{1/2}\right]\right\} \tag{15}$$

where E_1 and F_1 are the complete elliptic integrals. Both $K_1'(z)$ and $S_1(z)$ are tabulated in Table 1 [part of $S_1(z)$ data was obtained previously by Lighthill[6]].

Having obtained the source distribution of the streamtube in terms of the disturbances on the streamtube, we may find the F function of this streamtube in terms of these disturbances. By substituting Eq. (12) into the nondimensional form of Eq. (1), we obtain

$$\frac{F(\bar{y})}{(\alpha R)^{1/2}} = \int_0^{\bar{y}} w(\bar{t})T(\bar{y} - \bar{t})d\bar{t} \tag{16}$$

with

$$T(z) = \frac{1}{z^{1/2}} - \int_0^z \frac{S(z - z_1)}{z_1^{1/2}}dz_1 \tag{17}$$

Since $w(\bar{t})$ and $g(\bar{t})$ are also related by Eq. (14), then

$$\frac{F(\bar{y})}{(2\alpha R)^{1/2}} = g(0)U(\bar{y}) + \int_0^{\bar{y}} g'(\bar{x})U(\bar{y} - \bar{x})d\bar{x} \tag{18}$$

where

$$U(\bar{y}) = \frac{1}{\pi}\int_0^{\bar{y}} \frac{T(\bar{y} - \bar{t})}{\bar{t}^{1/2}}d\bar{t} = 1 - \int_0^{\bar{y}} S(z)dz \tag{19}$$

With $U(0) = 1$. By integrating Eq. (18) by parts and then by substituting Eq. (19), we obtain

$$\frac{F(\bar{y})}{(2\alpha R)^{1/2}} = g(\bar{y}) - \int_0^{\bar{y}} g(\bar{t})S(\bar{y} - \bar{t})d\bar{t} \tag{20}$$

In the dimensional form, the F function is related to the pressure distribution $P(x)$ in the form,

$$\frac{F(x)}{(2\alpha R)^{1/2}} = P(x) - \frac{1}{\alpha R}\int_0^x P(t)\, S\left(\frac{x - t}{\alpha R}\right)dt \tag{21}$$

Similarly, by substituting Eq. (13) into Eq. (1), we may express the F function in terms of the streamline deflection R' on the streamtube,

$$\frac{\alpha F(x)}{(2\alpha R)^{1/2}} = R'(x) - \frac{1}{\alpha R}\int_0^x R'(t)S_1\left(\frac{\alpha R}{x - t}\right)dt \tag{22}$$

Flow disturbances on the streamtube may also be expressed in terms of the corresponding F function by solving integral equations (1) and (22), respectively,

$$\frac{1}{\gamma M_\infty^2}\frac{\Delta p}{p_\infty} = -u = \frac{1}{(2\alpha R)^{1/2}}\left[F(x) + \frac{1}{\alpha R}\int_0^x F(t)K'\left(\frac{x - t}{\alpha R}\right)dt\right] \tag{23}$$

and

$$v = \frac{\alpha}{(2\alpha R)^{1/2}}\left[F(x) + \frac{1}{\alpha R}\int_0^x F(t)K_1'\left(\frac{x - t}{\alpha R}\right)dt\right] \tag{24}$$

Now Eqs. (23) and (24) [or Eqs. (21) and (22)] are exact relations between the flow disturbances and the F function on the streamtube ($r = R$). Since the radius R of the streamtube was chosen arbitrarily, these relations are valid at any distance from the body. Consequently, x and R in Eqs. (23) and (24) may be replaced by y and r, respectively. y is now determined by an exact characteristic equation,

$$x = \alpha(r - R) + y - kF(y)[r^{1/2} - R^{1/2}] - \int_0^y F(t)H\left(\frac{y - t}{\alpha r}; R\right)dt \tag{25}$$

with

$$H\left(\frac{y - t}{\alpha r}; R\right) = \frac{M_\infty^2}{2^{1/2}\alpha^{1/2}}\int_R^r \left[K_1'\left(\frac{y - t}{\alpha r}\right) + \frac{(\gamma - 1)M_\infty^2 + 2}{2\alpha^2}K'\left(\frac{y - t}{\alpha r}\right)\right]\frac{dr}{r^{3/2}} \tag{26}$$

Here, R is the local radius of a slender body or the mean radius of a particular streamtube. Now the F function at any location r may be obtained from a known F function at $R(R < r)$ by relating their arguments by Eq. (25)[1,4]. The F function at r is generally not a single valued function at certain regions of its argument. These multiple valued regions are remedied by the presence of shock waves. The flow disturbances at r may be determined from the local F function obtained by the exact relations (23) and (24) with R replaced by r.

By comparing the exact relations (23–25), with Whitham's asymptotic relations (2) and (3), it is observed that the last terms in Eqs. (23–25) are modifications of Whitham's relations, and these terms vanish at large distances. It is also interesting to note that these modification terms may be expanded in power series of $[(y - t)/r]^{1/2}$, and this series expansion is consistent with that suggested by Whitham.[4]

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Electron Quench Effects of SF_6 in Air and Argon Plasmas

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Introduction

Since sulfur hexafluoride (SF_6) has been shown to be effective for reducing electron concentrations in a plasma,[1,2,3] an equilibrium thermochemical study was made on

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